

Proof mutual exclusion of the ping-pong example

1) label the statements, so that we can reference to "before CS (Critical Section)"

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while(true) {  
  A1, B1: non_critical section  
  A2, B2: while(!(signal.turn == myid)) { }  
  A3, B3: critical section  
  A4, B4: signal.turn = (myid == 0) ? 1 : 0  
}
```

2) Define the invariant(s)

- (i) at(A3) -> term == 0
- (ii) at(B3) -> term == 1
- (iii) not[at(A3) AND at(B3)]

3) Proof the (i) -- (iii)

Proof (i)

at(A1): condition (i) is false => do not care about signal
at(A2): condition (i) is false => do not care about signal
at(A3): condition (i) is true => signal == 0, follows from the fact
that signal was 0 at(A2) AND the transition from A2->A3 did not
change value of signal
at(A4): condition (i) is false ==> do not care about signal

Now, we consider:

at(B1) : no change to signal
at(B2) : no change to signal
at(B3) : no change to signal
at(B4) : changes signal to 0
=> Invariant 1 is true

Proof (ii)

at(B1): condition (ii) is false => do not care about signal
at(B2): condition (ii) is false => do not care about signal
at(B3): condition (ii) is true => signal == 1, follows from the fact
that signal was 1 at(B2) AND the transition from B2->B3 did not
change value of signal
at(B4): condition (ii) is false ==> do not care about signal

Now, we consider:

at(A1) : no change to signal
at(A2) : no change to signal
at(A3) : no change to signal
at(A4) : changes signal to 1
=> Invariant 2 is true

For now we have shown that (i) and (ii) are TRUE

Proof(iii) (by contradiction)

Assume thread A entered CS (A3) at time t_1

Assume thread B entered CS (B3) at time t_2 , where $t_2 = t_1 + \delta$

--> CONTRADICTION: since we are in A3 signal MUST be 0 (cannot be 0 and 1 at the same time)

Assume thread B entered CS (B3) at time t_1

Assume thread A entered CS (A3) at time t_2 , where $t_2 = t_1 + \delta$

--> CONTRADICTION: since we are in B3 signal MUST be 1 (cannot be 0 and 1 at the same time)